Modeling and motion control of a hybrid-driven underwater glider

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Present study consists a mathematical model and motion control analysis for a hybrid-driven underwater glider, which can be propelled by using buoyancy and propeller with an addition of wings and a rudder that can be controlled independently. Thus, it can overcome the constraints of speed and maneuverability that was normally possessed by the fixed-winged buoyancy-driven underwater glider. Mathematical model of the glider is based on the Newton-Euler approach, and the hydrodynamics of the glider are estimated based on the Slender-body theory. Glider is controlled by six control inputs: the deflection angle of the right and left wing, the angle of a rudder, two net forces of a sliding mass and the pumping rate of a ballast pump. A Linear Quadratic Regulator (LQR) controller is used to obtain better control performance over the glider motion. Results show that the glider is stable, and the controller performance is satisfactory.

[Keywords: Underwater glider, Modeling, Motion control, LQR]

Introduction

The hybrid-driven underwater glider is a new class and practical underwater platform which can be a powerful tool for oceanographic sensing and sampling. The hybrid-driven glider combines the attributes of the buoyancy-driven underwater glider and conventional autonomous underwater vehicle (AUV). The development of this glider able to overcome the weaknesses of the conventional glider. The conventional underwater gliders such as the legacy underwater glider\(^1\)\(^-\)\(^4\) are fixed-winged buoyancy-driven underwater gliders, which have already demonstrated highly energy efficient. However, these gliders have limitations in terms of speed and maneuverability due to the absence of a propeller and limited external control surfaces such as controllable wings and a controllable rudder. Thus, in order to increase the efficiency of the conventional glider such as the speed and
maneuverability, the hybrid-driven underwater glider has been designed by having wings and a rudder that can be controlled independently and can be propelled by using buoyancy and propeller system.

The research works on the hybrid-driven underwater glider, as well as, the underwater glider with independently controllable wings were carried out by several researchers\textsuperscript{5-7}. However, the glider design and configuration that was simulated by these researchers were different from our design. Some of them has fixed wings and a controllable rudder\textsuperscript{5}, and some of them has no wings\textsuperscript{6}. Other than that, there was a work that presented a buoyancy-driven underwater glider with controllable wings\textsuperscript{7}.

Various control methods have been proposed to control underwater vehicles, whether through simulation or actual experiment\textsuperscript{8-9}. In terms of the glider controller, most of the existing gliders have used simple PID\textsuperscript{10-11}, and LQR\textsuperscript{12-14} controller to control the glider motion and attitude, and the performance comparison between PID and LQR for the glider control has been analyzed by Noh, Arshad and Mokhtar\textsuperscript{15}. However, these control systems are implemented for the Single-Input-Single-Output (SISO) system, and the presence of disturbance is neglected. Since, the hybrid-driven underwater glider has six control inputs; therefore, the motion control system is based on the Multiple-Input-Multiple-Output (MIMO) system and the LQR controller is used to control the glider motion. In addition, the presence of disturbance from the water currents has been taken into account. This paper explains the mathematical model and motion control of the hybrid-driven underwater glider. Hybrid-driven underwater glider in this work is an extension of the buoyancy-driven underwater glider and propeller-driven underwater glider presented in Isa and Arshad\textsuperscript{16-17}. Present study is an attempt to analyze the motion control of the glider by using the LQR controller and to compare the LQR performance between the glider motion with disturbance and without disturbance.
**Materials and Methods**

This section discusses the glider structure, mathematical model and equations of motion of the glider, and the hydrodynamics forces. In this work, the model derivation presented by Fossen\textsuperscript{18,19} and Graver\textsuperscript{20} is used as a basis for modeling the hybrid-driven autonomous underwater glider.

The USM hybrid-driven underwater glider has been modeled by using the Newtonian theory and the glider hydrodynamics have been analytically estimated based on the Strip theory approach\textsuperscript{21}. Glider is divided into two structures: the external structure and internal structure. On one hand, the external structure is divided of two areas: the wing-body area and tail area. Wing-body area is composed of independently controllable wings, a cylindrical hull and a nose. Meanwhile, the tail area has horizontal wings/fins, a controllable rudder and a propeller. On the other hand, the internal structure of the glider is composed of a ballast pump, a sliding mass, batteries, and electronics components such as the acoustic sensor, accelerometer, data logger and microcontroller. Figure 1 and 2 show the glider structure and configuration, respectively.

In order to make the glider has the ability to maneuver, the independently controllable wings and a rudder must have their own fixed frame. These external actuators able to rotate about its y-axis with respect to the body-frame, and the rotation angle in counter-clockwise is denoted as $\theta$. Thus, the rotation matrix of the right wings, $R_{rw}$, left wings, $R_{lw}$, and rudder, $R_{p}$, are defined as:

\begin{align}
R_{rw} &= \begin{bmatrix}
\cos \theta_{rw} & 0 & \sin \theta_{rw} \\
0 & 1 & 0 \\
-\sin \theta_{rw} & 0 & \cos \theta_{rw}
\end{bmatrix} \\
R_{lw} &= \begin{bmatrix}
\cos \theta_{lw} & 0 & \sin \theta_{lw} \\
0 & -1 & 0 \\
\sin \theta_{lw} & 0 & \cos \theta_{lw}
\end{bmatrix} \quad \text{and} \\
R_{p} &= \begin{bmatrix}
\cos \theta_{p} & 0 & \sin \theta_{p} \\
-\sin \theta_{p} & 0 & \cos \theta_{p}
\end{bmatrix}
\end{align}
In order to analyze the glider motion, the nonlinear equations of motion for the hybrid-driven underwater glider is written as:

\[ \eta = [x, y, z, \theta, \phi, \psi] = J(\eta) \dot{V} \quad \text{and} \]
\[ \dot{V} = [\dot{u}, \dot{v}, \dot{w}, \dot{\alpha}, \dot{\beta}, \dot{\gamma}] = \dot{V}_c + M^{-1}(-C(V) - D(V, \theta) - g(\eta) - P) \]

where \( J(\eta) \) is the rotation and transformation matrix of Euler angles, \( V \) is the translational and rotational velocity, \( V_c \) is the current velocity, \( M \) represents the system inertia matrix of the glider, \( C(V) \) represents the Coriolis-centripetal of the glider, \( D(V, \theta) \) is the damping forces and moments, \( g(\eta) \) is the gravitational and buoyancy forces and moments, and \( P \) represents the propeller forces.

The rotation and transformation matrix of Euler angles, \( J(\eta) \), is defined as:

\[ J(\eta) = \begin{bmatrix} R\theta & 0_{3 \times 3} \\ 0_{3 \times 3} & T_{\phi} \end{bmatrix} \]

where \( R\theta \) is the rotation matrix and \( T_{\phi} \) represents the transformation matrix. The translational and rotational velocity is denoted as:

\[ V = [v, \omega]^T \]

where \( v = [u, v, w]^T \) is the translational velocity and \( \omega = [\alpha, \beta, \gamma]^T \) is the rotational velocity.

However, since the glider can be propelled by buoyancy system, the parameter of \( \dot{q} \) in the equations (5) need to be rewritten. Thus, by referring to Graver\(^{20} \), the following equations which represent the effects of the sliding mass on \( \dot{q} \) is included in the equations of motion.

\[ \dot{q} = \frac{1}{I_s} ((M_s - M_2)uw - (r_{px}p_{px} + r_{pz}p_{pz})q - m_p g (r_{px} \cos \theta + r_{pz} \sin \theta) + M_{DL} - r_{px} \mu_x + r_{pz} \mu_z) \]

\[ \dot{r}_{px} = \frac{1}{m_p} p_{px} - u - r_{px} \theta \]

\[ \dot{r}_{pz} = \frac{1}{m_p} p_{pz} - w + r_{pz} q \]

\[ p_{px} = u \]
where $M_2$ and $M_3$ are the element of the system inertia matrix, $m_p$ is the internal sliding mass, $r_{p3}$ and $r_{p2}$ are the position of the sliding mass in the $x$ and $z$-direction, $p_{p3}$ and $p_{p2}$ represent the force of the sliding mass in the $x$ and $z$-direction, $m_b$ is the ballast pump mass, and $M_{D2}$ is the viscous moment.

**System Inertia Matrix**

The system inertia matrix of the rigid body and added mass, $M$, is defined as:

$$M = M_{RB} + M_A$$

The rigid-body system inertia matrix, $M_{RB}$, is defined as follows:

$$M_{RB} = \begin{bmatrix} m_{rb}(I_{orb}) & -m_{rb}g_x^2 \\ m_{rb}g_x^2 & I_{o} \end{bmatrix}$$

where $m_{rb}$ represents the mass of rigid body, $I_{o}$ represents the inertia matrix about the origin and $g_x$ represents the skew-symmetry matrix of the centre of gravity location with respect to the origin.

The added mass system inertia matrix, $M_A$, is denoted as:

$$M_A = \begin{bmatrix} -X_d & 0 & 0 & 0 & -y_dX_d \\ 0 & -Y_d & 0 & z_dX_d & 0 \\ 0 & 0 & -Z_d & y_dZ_d & x_dZ_d \\ -y_dX_d & 0 & x_dZ_d & -x_dY_d & 0 \\ y_dX_d & -x_dY_d & 0 & -x_dZ_d & -M_d \end{bmatrix}$$

**Coriolis and Centripetal Forces**

The Coriolis-centripetal of the rigid body and added mass, $C(V)$, is formulated as:

$$C(V) = C_{RB}(V)V + C_A(V_c)V_c$$

where $V_c$ is the relative velocity, which is defined as $V_c = V - V_c$. The $C_{RB}$ represents the Coriolis of rigid body and $C_A$ represents the Coriolis of added mass. Both of these notations are defined as:

$$C_{RB}(V) = \begin{bmatrix} 0_{3x3} & -m_{rb}V - m_{rb}g_x^2 \\ -m_{rb}g_x^2 & -I_{o} \end{bmatrix}$$
Damping Forces and Moments

According to Fossen\(^1\), the damping forces and moments, \( \mathbf{D}(\mathbf{V}_p) \), are defined as:

\[
\mathbf{D}(\mathbf{V}_p) = \mathbf{D} + \mathbf{D}_n(\mathbf{V}_p)
\]  \( \quad (20) \)

where \( \mathbf{D} \) is the linear damping matrix and \( \mathbf{D}_n(\mathbf{V}_p) \) is the nonlinear damping matrix. Both of these notations are defined as:

\[
\mathbf{D} = \begin{bmatrix}
-X_u & 0 & 0 & 0 \\
0 & -Y_v & 0 & 0 \\
0 & 0 & -Z_w & 0 \\
-x_{cg} \mathbf{K}_u & -y_{cg} \mathbf{K}_v & -z_{cg} \mathbf{K}_w \\
y_{cg} \mathbf{K}_u & -x_{cg} \mathbf{K}_v & -y_{cg} \mathbf{K}_w
\end{bmatrix}
\]  \( \quad (21) \)

\[
\mathbf{D}_n(\mathbf{V}_p) = \begin{bmatrix}
-X_{u|u}|u| & 0 & 0 & 0 \\
0 & -Y_{v|v}|v| & 0 & 0 \\
0 & 0 & -Z_{w|w}|w| & 0 \\
-x_{cg} \mathbf{K}_{u|u}|u| & -y_{cg} \mathbf{K}_{v|v}|v| & -z_{cg} \mathbf{K}_{w|w}|w| \\
y_{cg} \mathbf{K}_{u|u}|u| & -x_{cg} \mathbf{K}_{v|v}|v| & -y_{cg} \mathbf{K}_{w|w}|w|
\end{bmatrix}
\]  \( \quad (22) \)

Restoring Forces and Moments

The gravitational and buoyancy forces and moments, \( \mathbf{g}(\mathbf{q}) \), are defined as:
These forces are occurred due to weight, $W$, and buoyancy, $B$. The weight and buoyancy are defined as:

$$W = m_r g \quad \text{and} \quad B = \rho g V,$$

where $g$ is the gravity, $\rho$ is the water density and $V$ is the volume of the glider body. It is convenient to design the underwater glider with positive buoyancy ($B > W$), so that the glider will surface automatically in the case of an emergency situation such as power failure.

**Motion Controller Design**

The hybrid-driven underwater glider controls its motion and attitude through the external actuators (wings and a rudder) and internal actuators (a sliding mass and ballast pump). Thus, six control inputs are required. The control inputs are denoted as:

$$u = [\Delta r_w, \Delta r_v, \Delta p, u_x, u_z, u_b]^T$$

where $u_x$, $u_z$, and $u_b$ are the net force acting on sliding mass in $x$-direction, the net force acting on sliding mass in $z$-direction and the ballast pump rate, respectively.

In order to control the glider motion by using LQR, the nonlinear equations of motion were linearized about two operating points: with disturbance from the water currents and without disturbance. Table 1 shows the operating points.

This linearization produced the state-space of the complete linearized model, which consists of 18 inputs, 17 states and 17 outputs. However, for the motion control analysis, 6 inputs ($\Delta r_w$, $\Delta r_v$, $\Delta p$, $u_x$, $u_z$ and $u_b$), 14 states and 14 outputs were selected. The first three positions, $\eta_i$ i.e. $x$, $y$ and $z$ in
the states and outputs are neglected because it will not affect the dynamic of the glider. During the linearization process, the glider's velocity, $V$, is assumed as 2 m/s, and the propeller forces, $P$, are defined as 35.59 N, based on our propeller specification. Furthermore, the velocities of the water currents are considered as an unmeasured disturbance because there is no sensor that can be used to measure the velocities of water current. Thus, the velocities of the water current, $V_c$, in the $x$-direction are assumed as 0.5 m/s with a condition that $V_c < V$.

The motion controller was designed by using LQR so that the system will be asymptotically stabilized. The LQR controller is used because of its ability to handle the MIMO system through the state-space representation. LQR is a standard optimal control design that stabilizes control law by minimizing a cost function. The cost function is defined as:

$$J_{LQR} = \int_0^\infty x(t)^TQx(t) + \rho u(t)^TRu(t)dt$$

(26)

where $Q$ and $R$ are weighting matrices or a symmetric positive definite matrix for the state variables and the input variables, respectively, and $\rho$ is a positive constant. LQR controller can easily be designed by using a linear state feedback with a gain matrix, $K$, i.e.,

$$u(t) = -Kx(t).$$

(27)

In order to derive the LQR, the glider plant is assumed to be written in state-space form $x = Ax + Bu$, and that all of the $n$ states $x$ are available for the controller. Feedback gain is a matrix $K$, implemented as mentioned in (27). Thus, the system dynamics are then written as:

$$\dot{x} = (A - BK)x + BKx_d.$$

(28)

where $x_d$ represents the vector of desired states, and serves as the external input to the closed-loop system.

Then, the $L$ parameter of the LQR is calculated as:

$$L = \frac{1}{2}x^TQx + \frac{1}{2}u^TRu.$$ 

(29)
where the requirement that \( L \geq 0 \) implies that both \( Q \) and \( R \) are positive definite. Thus, in the case of linearized plant dynamics, the \( L_x = x^T Q, L_u = u^T R, f_x = A, \) and \( f_u = B, \) so that
\[
\dot{x} = Ax + Bu,
\]
(30)
\[
x(t_0) = x_0.
\]
(31)
\[
\lambda = -Qx - A^T \lambda, \lambda(t_f) = 0, \text{ and}
\]
(32)
\[
Ru + B^T \lambda = 0.
\]
(33)

By defining \( \lambda = P x, \) where \( P \) can be found by solving the continuous time Riccati differential equation, and inserting this into the equation (32), and then using the equation (30), and a substitution for \( u, \) the following matrix Riccati equation is obtained as:
\[
PAx + A^T Px + Qx - PB R^{-1} B^T Px + \ddot{P} = 0.
\]
(34)

Thus, the steady-state of LQR is defined as:
\[
PA + A^T P + Q - PB R^{-1} B^T P = 0.
\]
(35)

and then \( K \) is derived from \( P \) using the following equation:
\[
K = R^{-1} (B^T P).
\]
(36)

In this work, the values of \( Q \) and \( R \) are defined as:
\[
Q = \text{diag}(\rho, 0, 0, 0, 0, 0, 0, 0, 0, 0) = C^T C, \text{ and}
\]
(37)
\[
R = \text{diag}(1, 0, 0).
\]
(38)

Thus, the following gain matrix, \( K \), is obtained as:

\[
K_{\text{win}} = \begin{bmatrix}
-0.5 & -3.4 & -0.3 & -0.8 & 0.7 & 1.1 & -0.5 \\
-0.5 & 3.4 & -0.3 & 0.8 & 0.7 & -1.1 & -0.5 \\
0.2 & 0 & 0.6 & 0 & -0.1 & 0 & -0.2 \\
0 & -0.4 & 0 & -0.2 & 0 & 0.3 & 0 \\
0 & 0.02 & 0 & 0.2 & 0 & -0.03 & 0 \\
0 & -0.3 & 0 & 0.02 & 0 & 0.4 & 0
\end{bmatrix}
\]
Results and Discussion

This section presents the simulation results of the LQR controller for the glider. Generally, the results show the LQR performance for both conditions, which are motion with disturbance and without disturbance. The simulation was programmed by using Matlab™ and Simulink™.

The control system was simulated with different values of control inputs for 200 seconds, which controlling the gliding motion (pitching), maneuvering motion (rolling and yawing) and combination of both. Table 2 shows the control inputs from the reference model that were used in the simulation.

In the first 50 seconds, the deflection angle of right wing was set to 15° (0.2618 radian) and for the second 50 seconds, the deflection angle rudder was set to -15°. In the third and fourth 50 seconds, the net force of sliding mass in the x and z-direction, and the pumping rate of the ballast pump were set to -30 g(cm/s²), -30 g(cm/s²), 20 g/s and 30 g(cm/s²), 30 g(cm/s²), -20 g/s, respectively.
Figures 3-6 show the glider motion and attitude that was controlled by the LQR controller based on the control inputs in Table 2. Motion that are depicted in the figures are rolling, yawing, downward and upward motion. In addition, a comparison between the performance of LQR controller with disturbance and without disturbance is also presented.

Figure 3 shows the feedback of the LQR controller for both conditions. The graph has shown that, after 100 seconds, the roll and yaw angle were not affected by the different values of the sliding mass net force and ballast pump rate. Although the difference between the performance of the LQR with disturbance and without disturbance was obvious, the LQR able to compensate the disturbance from the water current and stabilized the glider.

Figure 4 presents the LQR feedback of the translational velocities namely the surge, sway, and heave. The graph has shown that the controller response of surge velocity for both conditions was quite similar, but the settling time for the LQR with disturbance was faster than the LQR without disturbance. On the other hand, there was a big gap between the response of the sway and heave for both conditions.

The LQR response of the rotational velocities namely the roll rate, pitch rate, and yaw rate, is shown in Figure 5. The graph has shown that the feedback response of the roll and yaw rate for both conditions had a similar response and settling time. However, the response of the pitch rate for both conditions was difference due to the presence of disturbance that affected the pitch angle of the glider.

Figure 6 shows the feedback response of the sliding mass and ballast pump for both conditions. The position of the sliding mass in x-direction was affected by the presence of the disturbance from the water current velocity. This is because the sliding mass controls the pitch angle of the glider. However, the sliding mass force and the ballast mass converged to the desired sliding mass net force and ballast mass rate.
In summary, the amplitude values of the outputs that were obtained from the LQR feedback are tabulated in Table 3. In addition, the error of the LQR feedback between motion with disturbance and without disturbance is presented in Table 4. Data in this table shows that the LQR feedback of the pitch rate has produced the highest average of error rate with a value of 11.2267 for 200 seconds of simulation. On other hand, the yaw rate has produced the lowest average of error with a value of 0.

**Conclusion**

In conclusion, this paper presents the mathematical model and motion control of the hybrid-driven underwater glider. The objectives of this paper are to analyze the motion control of the glider by using the LQR controller and to compare the LQR performance between the glider motion with disturbance and without disturbance. Due to that, the nonlinear glider model has been designed and linearized at two operating points, where the presence of water current velocity as a disturbance was included in one of the operating points. The simulation was programmed by using Matlab™ and Simulink™. Several values of control inputs have been tested, and the simulations demonstrated that the results are acceptable, and the stability of the glider has been gained by the LQR for the motion with disturbance and without disturbance. However, the feedback responses for both conditions had glitches. Due to that, a biologically inspired control algorithm has been designing to overcome the drawbacks of the motion controller of the glider.

**Acknowledgment**

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References


12. Leonard, N.E., Graver, J.G., Model based feedback control of autonomous underwater gliders, 

13. Lei, K., Yuwen, Z., Hui, Y., Zhikun, C., MATLAB-based simulation of buoyancy-driven underwater glider motion, 


16. Isa, K., Arshad, M.R., Buoyancy-driven underwater glider modelling and analysis of motion control, 

17. Isa, K., Arshad, M.R., Propeller-Driven Underwater Glider Modelling and Motion Control, 


Table 1: Linearization operating points

<table>
<thead>
<tr>
<th>Parameter</th>
<th>With disturbance</th>
<th>Without disturbance</th>
</tr>
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<tbody>
<tr>
<td><strong>States</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td></td>
<td>$[0,0,0,0,0]^T$</td>
</tr>
<tr>
<td>$V$</td>
<td></td>
<td>$[2,0,0,0,0]^T$</td>
</tr>
<tr>
<td>$r_{px}, r_{pz}, P_{px}, P_{pz}, m_b$</td>
<td></td>
<td>$[0,0,0,0,0]^T$</td>
</tr>
<tr>
<td><strong>Inputs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{tw}, \sigma_{hw}, \sigma_r$</td>
<td></td>
<td>$[0,0,0]^T$</td>
</tr>
<tr>
<td>$u_x, u_z, u_b$</td>
<td></td>
<td>$[0,0,0]^T$</td>
</tr>
<tr>
<td>$V_c$</td>
<td></td>
<td>$[0.5,0,0,0,0]^T$</td>
</tr>
<tr>
<td>$P$</td>
<td></td>
<td>$[35.59,0,0,0,0,0]^T$</td>
</tr>
<tr>
<td>Control Inputs</td>
<td>50s</td>
<td>50s</td>
</tr>
<tr>
<td>---------------</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>$\phi_{1 \phi}$ (rad)</td>
<td>0.2618</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_{2 \phi}$ (rad)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\alpha_{\phi}$ (rad)</td>
<td>0</td>
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<tr>
<td>$u_x$ g(cm/s²)</td>
<td>0</td>
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</tr>
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<td>$u_z$ g(cm/s²)</td>
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<td>$u_{\phi}$ g(cm/s²)</td>
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<td>0</td>
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<tr>
<td>Outputs</td>
<td>Time</td>
<td>Amplitude</td>
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<td>---------</td>
<td>------</td>
<td>-----------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>With disturbance</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50s</td>
</tr>
<tr>
<td>Roll ((\theta)) (rad)</td>
<td>-0.0056</td>
<td>0.0056</td>
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<tr>
<td>Pitch ((\phi)) (rad)</td>
<td>-0.3539</td>
<td>0.3535</td>
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<tr>
<td>Yaw ((\psi)) (rad)</td>
<td>0.0662</td>
<td>-0.0662</td>
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<tr>
<td>Surge (u) (m/s)</td>
<td>-16.4955</td>
<td>16.4734</td>
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<tr>
<td>Sway (v) (m/s)</td>
<td>0.0122</td>
<td>-0.0122</td>
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<tr>
<td>Heave (w) (m/s)</td>
<td>-6.9038</td>
<td>6.9468</td>
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<tr>
<td>Roll rate (p) (rad/s)</td>
<td>-1.2918</td>
<td>2.5836e-09</td>
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<tr>
<td>Pitch rate (q) (rad/s)</td>
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<tr>
<td>Yaw rate (r) (rad/s)</td>
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<td>-1.7055e-06</td>
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<tr>
<td>Rpx (cm)</td>
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<tr>
<td>Ppx (g/(\text{cm/s}^2))</td>
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<td>16.4777</td>
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<td>Ppz (g/(\text{cm/s}^2))</td>
<td>-6.9044</td>
<td>6.9483</td>
</tr>
<tr>
<td>Mb (g)</td>
<td>24.3426</td>
<td>-24.3262</td>
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</table>

Table 3: LQR feedback of the outputs
### Table 4: Error of LQR Feedback between motion with disturbance and without disturbance

<table>
<thead>
<tr>
<th>Outputs\Time</th>
<th>Amplitude</th>
</tr>
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<td>Feedback error between motion with disturbance and without disturbance</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50s</td>
</tr>
<tr>
<td>Roll ((\phi)) (rad)</td>
<td>0.0038</td>
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<tr>
<td>Pitch ((\theta)) (rad)</td>
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<td>Yaw ((\psi)) (rad)</td>
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<td>Surge ((u)) (m/s)</td>
<td>1.0155</td>
</tr>
<tr>
<td>Sway ((v)) (m/s)</td>
<td>0.0252</td>
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<tr>
<td>Heave ((w)) (m/s)</td>
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<tr>
<td>Roll rate ((p)) (rad/s)</td>
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<tr>
<td>Pitch rate ((q)) (rad/s)</td>
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<tr>
<td>Yaw rate ((r)) (rad/s)</td>
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<tr>
<td>Rpx (cm)</td>
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<tr>
<td>Rpz (cm)</td>
<td>2.0226</td>
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<tr>
<td>Ppx g(cm/s(^2))</td>
<td>1.0362</td>
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<tr>
<td>Ppz g(cm/s(^2))</td>
<td>6.7356</td>
</tr>
<tr>
<td>Mb (g)</td>
<td>2.3020</td>
</tr>
</tbody>
</table>

**Figure 1:** The glider structure and its reference frame

**Figure 2:** The glider configuration

**Figure 3:** The controllers feedback of the Euler angles

**Figure 4:** The translational velocities feedback response

**Figure 5:** The rotational velocities feedback response

**Figure 6:** The controllers feedback of the sliding mass and ballast pump